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A New Concept of Filters for Biomedical Data Processing Needs

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This paper presents a new class of filters that can meet biomedical signal processing needs. The paper is written in a technical note style, therefore, the proposed filters are not discussed with respect to a specific problem appearing in processing of a particular biosignal. The class of filters presented in this note should be treated as a new effective tool which can be applied to many cases of biomedical signals, especially when the processing time is very important. Nevertheless, a simple example of biomedical signal filtering is presented. This paper presents a new concept of continuous-time Butterworth filters whose parameters are varied in time. Thanks to the variation of the filter parameters, the time-varying filter response is considerably faster in comparison with the traditional time-invariant filters. Therefore, we can measure and register a lot of details in the initial stage of signal duration, which is not possible in the case of traditional time-invariant filters due to their long-lasting transients. Results verifying the effectiveness of the proposed filters are presented and compared to the traditional time-invariant filter structures.

K e y w o r d s: signal processing, data smoothing, biomedical signals, transient state, time-varying systems

1. Introduction

In [1] Robertson et al. present some investigations over traditional Butterworth and critically damped filters. The review of traditional filters, which has been carried out in this paper is very useful. However, the advantages and disadvantages of these filters are well known, and their description was reported by Chen [2], Schaumann et al. [3], and Su [4]. Nowadays, there is a need for looking for a new filter structure which will be able to work as fast as possible. The most common filter responses are the Butterworth, Chebyshev, elliptic, and Bessel-Thomson types.

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The Butterworth filters are often chosen for smoothing biomedical data because their gain response is maximally flat in the passband, and the roll-off rate is at adequate level. In the time domain, the Butterworth low-pass filters are characterized by undesirable overshoots and quite long transients. The magnitude response of the low-pass Butterworth filter can be written as follows:

$$\left|H_F(j\omega)\right|^2 = \frac{h_0}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}} \tag{1}$$

where n is the filter order, h_0 is the DC gain (gain at zero frequency), and ω_c is the 3 dB limit frequency.

In many cases of biomedical signal analysis, there is a need for a data filtering. One of the main aims of the filtering is to smooth the data as fast as possible. This requirement is related to improvement of the filter properties in the time domain. The problem of improving the transient performance has been considered by Kaszynski et al. [5] to successfully reduce the time of processing of the brain average-evoked potentials.

The commonly used smoothing filters are, in many cases, useless due to their long-lasting transients and undesirable overshoots. For traditional time-invariant filters there are only small possibilities of transient reduction, since the filter parameters are calculated on the basis of the assumed approximation method. This fact guarantees that the frequency requirements are satisfied without taking into consideration the characteristic of the transient state. If the requirements on the frequency characteristic are imposed, we can slightly influence the shortening of the transient state duration of the n-th order filter by choosing different approximation methods. The uncertainty principle states that it is not possible to achieve a shorter rise time of a low-pass filter output signal when the filter passband is constant. However, one can obtain significant changes of the transient state duration by variation of the filter passband. This procedure is related to the change of the value of filter coefficients. Such a kind of technique has been used in success by Piskorowski et al. [6] for elimination of undesirable effects of the group delay compensation. The theory of linear time-varying continuous-time systems is well established and was widely described by Claasen et al. [7], Margrave [8], and Zadeh [9,10].

In this note, a new concept of the Butterworth filters whose parameters are varied in time is presented. Thanks to variation of the filter parameters, the time-varying filter response is considerably faster and free from overshoots in comparison with the traditional time-invariant filters. The outline of the paper is as follows.

In Section 2, the mathematical formulation of the time-varying low-pass filter is presented. Section 3 then presents the results of simulations carried out with the aid of Matlab-Simulink software. The conclusions are presented in Section 4.

2. Mathematical Formulation of Time-varying Filter

Dynamic properties of the second order low-pass filter (or filter of constant component) are described by the damping ratio ξ and the natural frequency ω_n. The transfer function of this filter can be written as follows:

$$H(s) = \frac{1}{\omega_n^{-2} s^2 + 2\xi\omega_n^{-1} s + 1}. \tag{2}$$

It is well known that the larger value of the natural frequency ω_n, the shorter transient of the filter. On the other hand, the larger value of the damping ratio ξ is, the smaller overshoot of the filter is. By changing these parameters in time, we can improve the dynamics of the filter and obtain significant reduction of the transient duration. This situation leads to a time-varying filter design.

The time-varying filter design is the result of modeling of the differential equation which describes the filter in the time domain. For the purpose of the filter response improvement it was assumed that dynamic parameters of the filter will be varied in time. Therefore, the model of the filter has the following form:

$$\omega_n^{-2}(t) y''(t) + 2\xi(t)\omega_n^{-1}(t) y'(t) + y(t) = x(t) \tag{3}$$

where $x(t)$ and $y(t)$ are the filter input and output, respectively. Moreover, $\omega_n(t)$ is a function of the natural frequency, and $\xi(t)$ is a function of the damping ratio.

In order to shorten the transient state of the filter, we have assumed (on the basis of computer simulations and previous investigations reported by Piskorowski et al. [6]) the functions of the filter parameters in the following form:

$$\omega_n(t) = d_\omega \bar{\omega}_n \left[1 - \frac{d_\omega - 1}{d_\omega} h(t)\right] \tag{4}$$

$$\xi_n(t) = d_\xi \bar{\xi} \left[1 - \frac{d_\xi - 1}{d_\xi} h(t)\right] \tag{5}$$

where $\bar{\omega}_n$ and $\bar{\xi}$ are the natural frequency and the damping ratio, which come from the Butterworth approximation. The coefficients d_ω and d_ξ are the variation ranges of the functions $\omega_n(t)$ and $\xi(t)$. These parameters are described by the following ratios:

$$d_\omega = \frac{\omega_n(0)}{\bar{\omega}_n}, \qquad d_\xi = \frac{\xi(0)}{\bar{\xi}}. \tag{6}$$

The choice of the form of the functions (4) and (5) was connected with the easiness of the generation in the analogue technique. The function $h(t)$ in (4) and (5) describes the step response of the second order supportive system $H_s(s)$ which has the following form:

$$H_s(s) = \frac{1}{\omega_{nf}^{-2} s^2 + 2\xi_f \omega_{nf}^{-1} s + 1}. \tag{7}$$

Therefore the step response $h(t)$ of $H_s(s)$ can be written as follows:

$$h(t) = \ell^{-1}\left(\frac{1}{s} H_s(s)\right) = \ell^{-1}\left(\frac{1}{s} \frac{1}{\omega_{nf}^{-2} s^2 + 2\xi_f \omega_{nf}^{-1} s + 1}\right) \tag{8}$$

where ℓ^{-1} is the inverse Laplace transform, and ω_{nf} and ξ_f are the natural frequency and the damping ratio of the second order supportive system.

The functions $\omega_n(t)$ and $\xi(t)$ should not possess oscillations in their run, so $\xi_f = 0.9$ was established. For $\xi_f < 1$, relation (8) can be written in the following form:

$$h(t) = 1 - \left[\cos\left(\omega_{nf} t \sqrt{1-\xi_f^{\,2}}\right) + \frac{\sin\left(\omega_{nf} t \sqrt{1-\xi_f^{\,2}}\right)}{\sqrt{1-\xi_f^{\,2}}}\right] \exp(-\xi\ \omega_{nf} t). \tag{9}$$

With respect to the functions (4) and (5), ξ_f can be called as the oscillation factor, and ω_{nf} as the variation rate of the functions $\omega_n(t)$ and $\xi(t)$.

The main assumption imposed on the functions $\omega_n(t)$ and $\xi(t)$ is the necessity of settling during the transient state of the original time-invariant filter. This condition can be written as

$$\forall_{t>t_{s\alpha}} \omega_n(t) = \bar{\omega}_n \pm \alpha, \qquad \forall_{t>t_{s\alpha}} \xi(t) = \bar{\xi} \pm \alpha \tag{10}$$

where $t_{s\alpha}$ is the settling time (with assumed accuracy $\alpha = 5\%$) of the original time-invariant filter.

The function $\omega_n(t)$ starts from a larger value than $\bar{\omega}_n$, which means that this function decreases ($d_\omega > 1$) in the variation interval $[0, t_{s\alpha}]$. Such a run of the function $\omega_n(t)$ shifts the cutoff frequency to a larger value in the initial phase of the filter work. The function $\xi(t)$ also starts from a larger value than $\bar{\xi}$, which means that this function decreases ($d_\xi > 1$) in the variation interval. Such a run of the function $\xi(t)$ causes stronger damping of the input signal in the initial phase of the filter work, and the suppression of undesirable overshoot in the step response.

Introduction of the time-varying parameters requires an examination of the stability of the systems with element containing varying parameters. Kaszynski [11] presents a proof of the stability, which is based on the second Lyapunov method. The analysis of this proof leads to the following stability conditions of the second order time-varying system

$$\omega_n(t) > 0 \tag{11}$$

$$\xi(t) > 0 \tag{12}$$

$$\left|\frac{d\omega_n(t)}{dt}\right| < \left|2\xi(t)\omega_n{}^2(t)\right| . \tag{13}$$

It follows from the relation (13) that for the second order system to be stable it suffices that the rate of changes of the natural frequency is bounded by the product of the functions $\omega_n(t)$ and $\xi(t)$. According to (11) and (12), the functions $\omega_n(t)$ and $\xi(t)$, have to be positive.

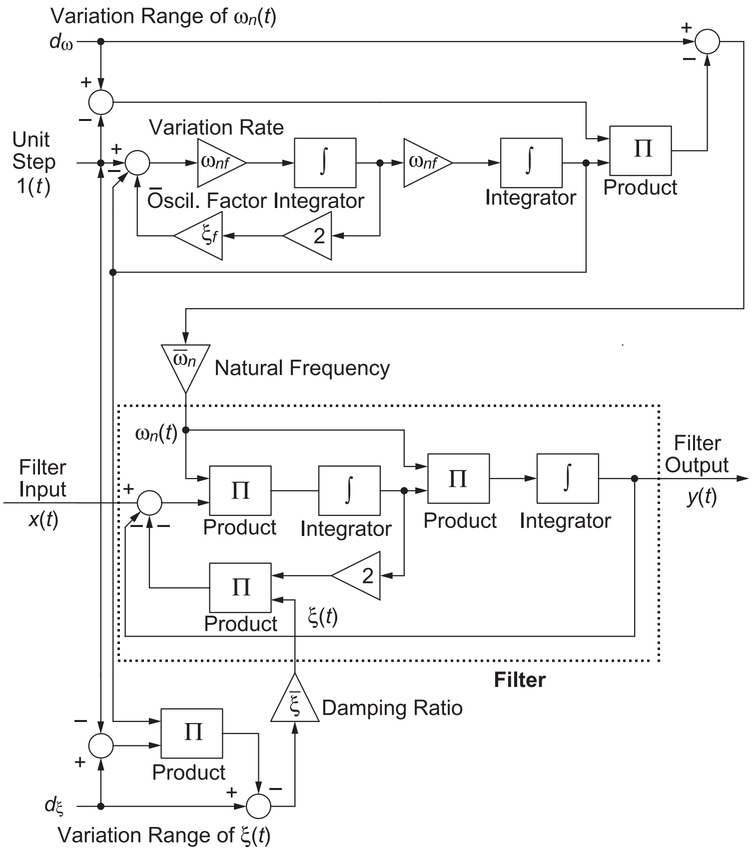

Fig. 1. Detailed model of the second order time-varying Butterworth filter

In this paper, only the low-pass filter type has been considered. However, it is easy to transform the low-pass filter to the other types of filters (i.e. high-pass, band-pass, and band-stop) using well known transformations [2–4]. After transforming, the filter should be decomposed to the first and the second order sections. Then, the natural frequencies and the damping ratios of these sections should be derived and varied according to the rules presented in this paper.

Figure 1 presents a detailed model of the second order time-varying Butterworth filter which has been discussed in this paper. A classical implementation of the time-varying approach described in this paper requires use of multipliers, adders, and two additional integrators. As one can notice, the complexity of the overall system underwent a significant increase. However, in situations, in which the transient should be as short as possible this complexity increase may be profitable.

3. Results of Simulations

In order to illustrate, how the time-varying principle influences the dynamics of a low-pass filter, the second order continuous-time Butterworth filter with cutoff frequency ω_c = 1 rad/s has been used. It is worth to add in this place that one of the simplest filters has been chosen for simulations in respect to the filter order and cutoff frequency. However, the time-varying principle which has been described in Section 2 can be applied for any filter order and cutoff frequency.

The transfer function of the above mentioned filter has the following form:

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}. \tag{14}$$

3.1. Overshoot Elimination

If the main aim of the transient improvement is to minimize the filter overshoot, the damping ratio ξ should be varied in time. It is well known that the larger damping ratio, the smaller overshoot. Using this principle, it is clear that the damping ratio should be larger in the initial phase of the filter work, so as to eliminate the overshoot from the filter response. For the second order Butterworth filter the variation range of the damping ratio function has been chosen as follows:

$$d_\xi = \frac{\xi(0)}{\bar{\xi}} = 1.5. \tag{15}$$

This values states that in the initial phase the damping ratio is 1.5 times larger than the one which follows from Butterworth approximation. The function $\xi(t)$ is shown in Fig. 2 and the step responses of the traditional and the time-varying filter are presented in Fig. 4. It is clearly seen that the response of the time-varying filter is free from overshoots.

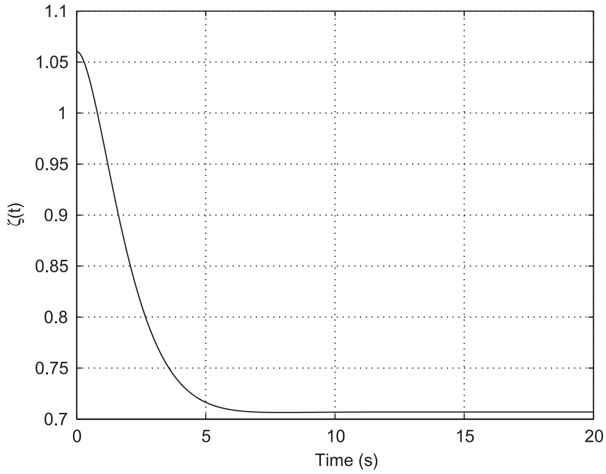

Fig. 2. Function $\xi(t)$

3.2. Transient Reduction

If the main aim of the transient improvement is to reduce the transient duration, the natural frequency ω_n should be varied in time. It is well known that the larger natural frequency, the shorter transient of the filter. Using this principle, it is clear that the natural frequency should be larger in the initial phase of the filter work. For the second order Butterworth filter the variation range of the natural frequency function has been chosen as follows:

$$d_\omega = \frac{\omega_n(0)}{\bar{\omega}_n} = 5\,. \tag{16}$$

This values states that in the initial phase the natural frequency is 5 times larger than the one which follows from Butterworth approximation. The function $\omega_n(t)$ is shown in Fig. 3 and the step responses of the traditional and the time-varying filter are presented in Fig. 4. It is clearly seen that the transient of the time-varying filter underwent a significant shortening.

3.3. Transient Reduction and Overshoot Elimination

If the aim of the transient improvement is to reduce the transient duration and simultaneously eliminate the overshoot from the filter response, both damping ratio and natural frequency should be varied in time according to the rules presented in the previous subsections. The results of the transient reduction and the overshoot minimization are presented in Fig. 4.

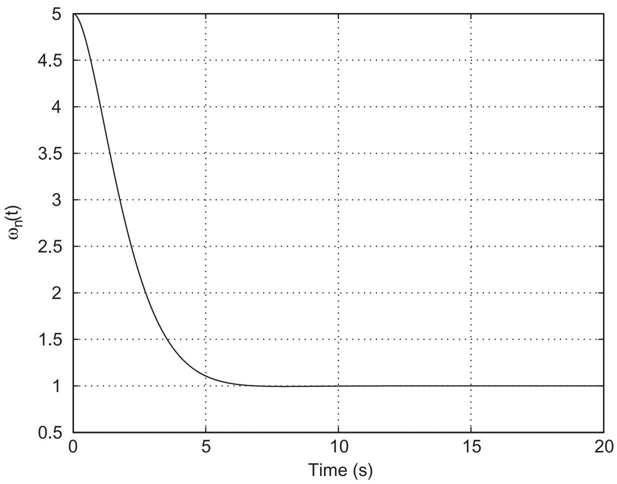

Fig. 3. Function $\omega_n(t)$

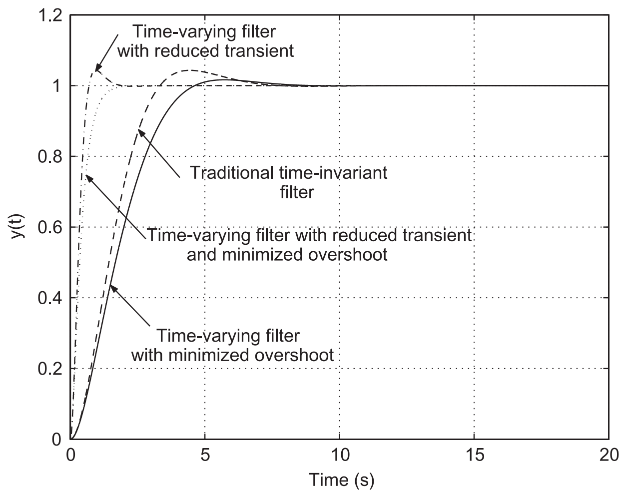

Fig. 4. Results of transient reduction and overshoot minimization

3.4. Example of Application

Time-varying continuous-time filters whose concept has been presented in this paper can be used for biomedical signal processing needs in twofold manner. Firstly, this kind of filters can be used when a filtering system is entirely analog. In such a system the proposed time-varying filter acts as the main filter which suppresses undesirable components of a signal which is to be processed.

Secondly, the time-varying continuous-time filter can be used as an anti-aliasing filter. Signal processing systems that appear to be entirely digital often contain one or more analog continuous-time filters. Anti-aliasing filters connect the real-world analog signals to the digital signal processor and provide band limiting before the signals can be sampled for further processing with sampled-data or digital techniques.

In this note, an example of biomedical signal processing in which the analogue time-varying filter acts as the main filtering system will be presented. Fig. 5(a) presents a sample motor unit action potential (MUAP) which has been distorted during recording. As one can notice, the resultant signal is the combination of the useful signal and an additive noise which should be suppressed. Figs. 5(b) and 6 present the result of filtering the using traditional time-invariant second order Butterworth filter with cutoff frequency $\omega_c = 10$ Hz and its time-varying equivalent. It is easy to notice, especially in Fig. 6 which shows the initial stage of the signal, that the time-varying filter is considerably faster than the traditional time-invariant one.

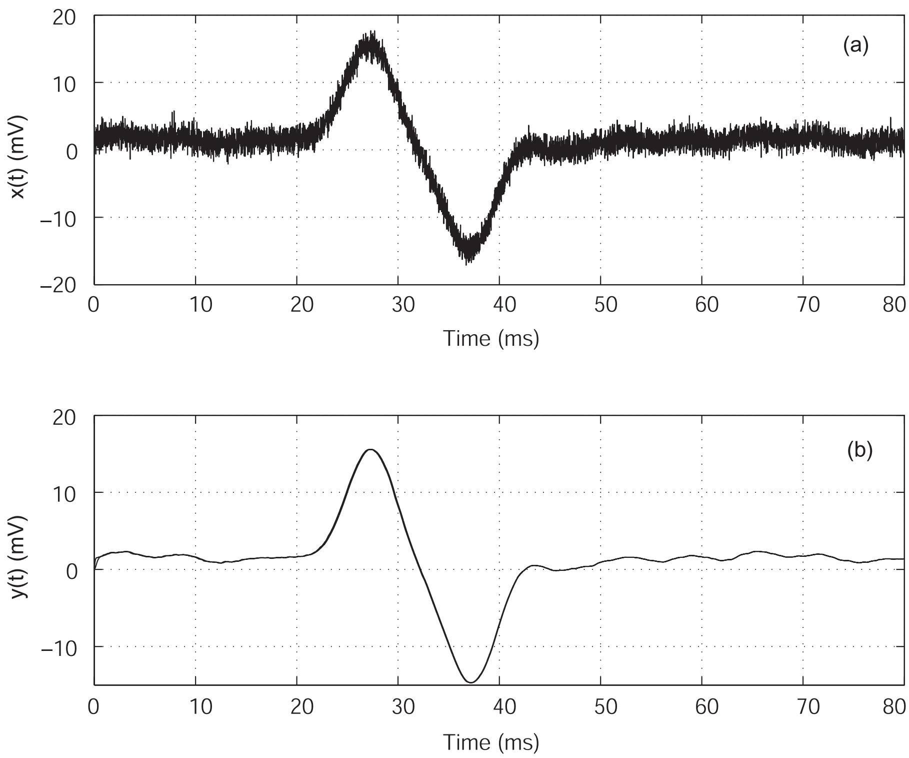

Fig. 5. (a) Motor unit action potential (MUAP) distorted by additive noise. (b) Motor unit action potential (MUAP) after filtering

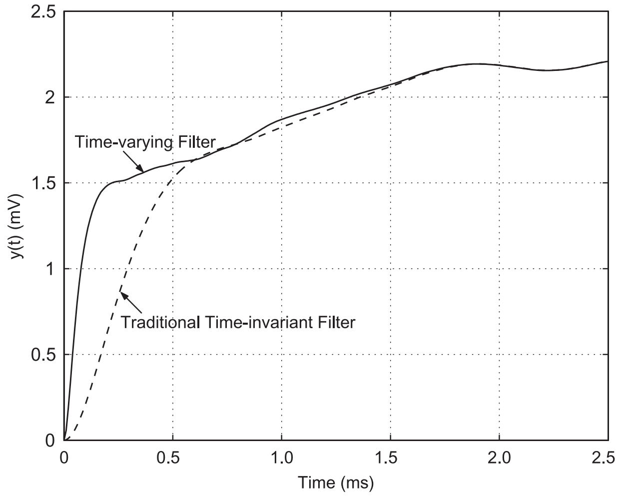

Fig. 6. Initial stage of the filtered MUAP signal. Time-invariant and time-varying filter response comparison

Therefore, using time-varying filters, we can measure and register a lot of details in the initial stage of signal duration, which is not possible in the case of traditional time-invariant filters due to their long-lasting transients.

4. Conclusions

As it has been proven, the introduction of time-varying coefficients to the low-pass Butterworth filter yields good results. By varying the filter parameters, it is possible to reduce the filter transient and eliminate the overshoot from the filter response. This fact may have a very important meaning in the case of the processing of many kinds of biomedical data. The behaviour of the time-varying filters has been presented with the aid of the step responses, similarly like in the technical note published by Robertson et al. [1]. Besides, a simple example of application to biomedical data has been also presented.

Summarizing, in the paper, a new concept of Butterworth filters whose parameters are varied in time has been presented. Thanks to variation of the filter parameters, the time-varying filter response is considerably faster and free from overshoots in comparison with the traditional time-invariant filters. Therefore, one can measure and register a lot of details in the initial stage of signals duration, which is not possible in traditional time-invariant filters due to their long-lasting transients. It seems that further examinations of time-varying filters when applied to the biomedical signal processing are needed.

The filter configuration presented in this paper can be implemented with the aid of the dynamic translinear technique which has been described by Mulder et al. [12] and Diepstraten [13]. By using the dynamic translinear principle, it is possible to implement linear and nonlinear differential equations, using transistors and capacitors only. Dynamic translinear circuits are excellently tunable across a wide range of several parameters, such as cut-off frequency, quality factor and gain, which increases their designability and makes them attractive to be used as standard cells or programmable building blocks. In fact, the dynamic translinear principle facilitates a direct mapping of any function, described by differential equations, onto silicon.

At the end of this paper, it is worth to add that the proposed filter structures can be easily transformed to digital filters. For that purpose, the continuous-time integrators from Fig. 1 should be transformed to their digital equivalents with the aid of the well known bilinear transform.

Acknowledgments

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