

Open Loop Hybrid Circulatory Model: The Effect of the Arterial Lumped Parameter Loading Structure on Selected Ventricular and Circulatory Variables

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Different combinations of the artero-ventricular coupling design (numerical, physical and hybrid) and the arterial system structure (four-element standard, simplified, modified and three-element three-lump “ladder” Windkessel) have been applied in an open loop circulatory model to test their influence on selected ventricular and circulatory variables.

Numerical investigations have shown that a four-element Windkessel with an introduced in series lumped inertance can evoke some numerical problems e.g. when combined with the simplified ventricular model containing “ideal” zero switching time heart valves or constant valve resistance during opening.

The four-element Windkessel structure modification i.e. replacing the in series inertance by the parallel one, considerably improves the network match. Also the three-element three-lump “ladder” Windkessel has been found very useful in the blood circulation modelling thanks to relatively small input inertance and high input capacitance of its first lump.

K e y w o r d s: hybrid numerical-physical model, lumped parameter model, Windkessel model, ventricular loading

1. Introduction

The basis of different methods to estimate the main elements of the heart load was formed on a two-element Windkessel model. The two-element model (RC) [1] consists of the total arterial compliance C , mainly determined by the elastic properties of the large arteries, first of all the aorta, treated as a single elastic chamber, and total

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peripheral linear resistance R_p , mainly located in capillaries and small arterial and venous vessels.

According to Stergiopoulos et al. [2], the two-element Windkessel model is not a good tool to reproduce the arterial input impedance Z_{in} (Z_{in} = oscillatory pressure/oscillatory flow) as it produces not realistic shapes of the aortic pressure wave [3] resulting from the zero inertia.

To overcome that weakness of the two-element Windkessel model, Westerhof et al. introduced the three-element Windkessel model [4]. To facilitate the analysis of the aortic-ventricular coupling, Burkhoff et al. [5] developed a simplified model of the aortic input properties also as the three-element Windkessel. The analysis of the obtained results indicated that most of the variables examined were not significantly different from those estimated with standard techniques. However, the major differences existed in the peak aortic flow and the mean arterial pressure.

Depending on the object of investigations, improvement of the basic Windkessel model proceeded in different directions. When physical and numerical models of the human blood circulation are constructed for different objectives, including interpretation of clinical observations and testing of prosthetic cardiovascular devices, it is important to validate quantitatively the dynamic response of the models in terms of the input impedance of the arterial system. To meet this need, Sharp and Dharmalingam [6] developed an improved physical model of the circulation. Using a computer study, they first identified the configuration of the lumped parameter elements in a model of the systemic circulation; the result was in a good match with the human aortic input impedance using a minimum number of elements. Numerical results showed that a three-element model with two resistors and the compliance (RCR) produced reasonable matching without undue complication. For high frequency, the RCR system provides an improvement in phase that approaches asymptotically to zero degrees while the phase of the simplest RC model approaches to -90° . The modulus of the RCR system also remains higher at high frequency, subsequently improving the match with human data.

Further increase of the number of elements in the ventricular loading system again results in some modulus and phase improvements that, according to Dharmalingam [6], are small. Stergiopoulos et al., in their later studies [7] found that the three-element Windkessel, although an almost perfect load for isolated heart studies, does not lead to accurate estimates of some arterial variables. To overcome this problem, they introduced an inertial term in parallel with the characteristic resistance. From the experiments in seven dogs they found that the three-element Windkessel overestimated total arterial compliance, whereas, the four-element Windkessel compliance estimates were not different from values derived from the data obtained with the standard techniques. Also ascending aortic pressure could be better predicted from aortic flow by using the four-element Windkessel than the three-element Windkessel. Moreover, the characteristic resistance was underestimated using the three-element Windkessel, whereas the four-element

Windkessel estimation differed marginally from the averaged impedance modulus at high frequencies. When applied to the human, the four-element Windkessel also was more accurate in these same aspects.

It is necessary to point out that some numerical problems in functioning of the circulatory model during digital simulation and in hybrid configuration are quite realistic. The mechanisms underlying these problems can be connected with specific operation of the current source (simulating the numerical aorta) bringing about aortic pressure and flow oscillations. Also presence in a model of connected in series lumped inductance (inertance), even relatively small, can evoke oscillations when combined with some simplifications of the ventricular model e.g., “ideal” heart valves, zero switching time and constant valve resistance during opening. It implies that the heart output impedance varies instantaneously from relatively small to infinite value, resulting in “numerically” infinitive acceleration of fluid and infinitive valve pressure drop.

Different combinations of the artero-ventricular coupling design have been applied in the open loop circulatory model to test their influence on selected ventricular and circulatory variables. Also the effect of the arterial system structure on ventricular and aortic pressure has been examined.

2. Materials and Methods

2.1. A Hybrid (Physical-numerical) Model-structure and Design

When forming hybrid (physical-numerical) models of the blood circulation or their functional modules, the same methodology has been applied based on the following sequence of tasks:

- Developing the mathematical model of the circulatory system or its fragment according to the planned application.
- Defining the adequate numerical model i.e. the set of differential and algebraic equations to create its physical analog in the form of the network of the lumped-parameter electrical elements (R, L, C) and discrete elements (diodes and controlled sources). An electrical equivalent of a basic closed loop model of blood circulation has been presented in Fig. 1. The circulatory system in this figure is divided into seven main sections, including coronary circulation. Each section can be easily modified or replaced.
- Defining a physical part of the system indispensable to perform certain investigations e.g. heart assistance using an Intra-Aortic Balloon Pump (IABP) or Left Ventricular Assist Device (LVAD).
- Choosing the structure of a hybrid model, with the physical (electrical) part, isolated from the numerical model of the circulatory system; it is identical with the structure of the planned in-coming hydraulic part. The electro-numerical model allows selecting working parameters of the A/D and D/A transformers to couple

a physical (electrical or hydraulic) environment with a numerical environment of the computer control program (Fig. 2).

- Designing a hydro-numerical or hydro-electro-numerical model of the blood circulation; it can be done replacing some electrical elements of the electro-numerical model by their functional hydraulic equivalents e.g. electrical diodes by artificial heart valves, electrically controlled current sources by electrically controlled flow sources and so on. In the hydro-numerical model, to solve the equations of the mathematical model and to operate the A/D and

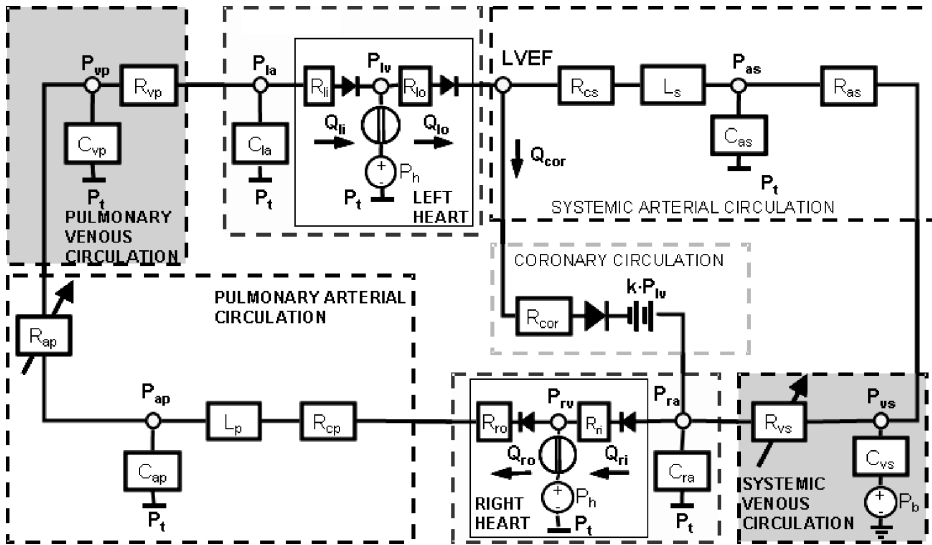


Fig. 1. Electric diagram of the basic closed loop circulatory model

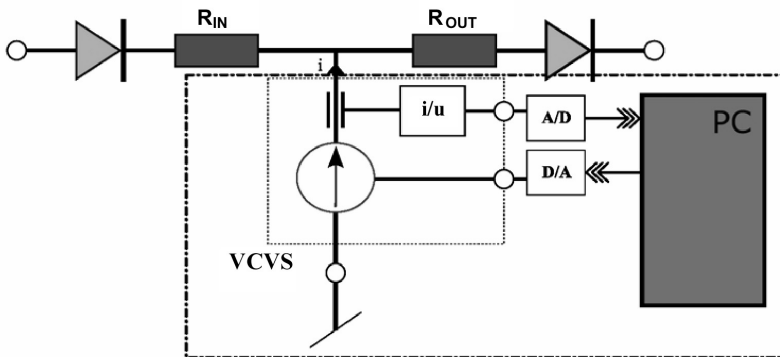


Fig. 2. A hybrid (electro-numerical) model of the left ventricle — general structure. R_{IN} , R_{OUT} — input (mitral), output (aortic) heart valve resistance, VCVS — voltage controlled voltage source

D/A transformers, the computer program and a certain part of the software remain unchanged.

The structure of a model is strictly dependent on application and is usually a compromise between the cost and complexity.

2.2. A Hybrid Electro-numerical Model of the Left Ventricle

According to the need, any physical part of a hybrid model can be based on electrical or hydraulic structures. The choice of the electrical structure depends on application and is founded on the electro-hydraulic analogy: conversion of pressure to voltage, flow to current and volume to electrical charge.

For many applications, most important is numerical and/or physical modeling of the left ventricle (LV) and the right ventricle (RV). The development of pure physical models of the ventricles, and of the circulatory system as a whole, is limited by their cost and mechanical characteristics e.g. the ventricular variable elastance [8–14].

In many cases, numerical models can be used to reproduce accurately complex mechanical characteristics of the ventricle and the circulatory system but these models cannot be directly connected to some physical medical devices, such as IABP or LVAD, without a physical section. It should be noted nevertheless that the section of the model that needs a physical representation is limited e.g. to the area of the balloon insertion or the blood inlet and outlet. Basing on these considerations, a new class of circulatory models defined as hybrid was proposed in the last years [15–17]. These models are composed of numerical and physical sections based indifferently on hydraulic or electrical structures. The existing prototypes have been developed for specific applications. These solutions suffer from limitations due to the software environment and, what is more important, cannot be used in each hemodynamic conditions determined by the use of mechanical circulatory assistance.

The prototype of the left ventricle applied in the models discussed in this paper has been generally based on a numerical-electrical structure and it takes all advantages of the numerical and physical (hydraulic) models. It has been developed at the Institute of Biocybernetics and Biomedical Engineering PAN (Poland) in close co-operation with the Institute of Clinical Physiology CNR (Italy). The laboratory tests presented in the paper were performed on the ventricular model both in hybrid and in pure numerical configuration using an open loop system.

The prototypes of hybrid models developed till now [15–18] have been based on Real Time structure but they operate in WindowsTM environment what implies a number of disadvantages. The primary aim of this work was the application of the LabVIEWTM Real Time environment in hybrid modeling to extend its applications and increase accuracy and reliability.

3. Laboratory Testing of the Hybrid Circulatory Model in the Open-loop Configuration

3.1. Effect of the Artero-ventricular Coupling Design on Selected Left Ventricular and Systemic Variables

Three combinations of different design artero-ventricular coupling (numerical, physical and hybrid) have been verified in an open loop circulatory model to test their influence on left ventricular (p_{lv}) and aortic (p_{as}) pressures:

- Hybrid ventricle-electrical arterial model.
- Hybrid ventricle-numerical arterial model.
- Numerical ventricle-numerical arterial model.

Table 1. List of variables

Abbreviations	Variables
A/D, D/A	Analog-to-digital and digital-to-analog converter
C	Arterial compliance
C_{as}/C_{ap}	Systemic/pulmonary arterial compliance
C_{La}/C_{ra}	Left/right atrial compliance
C_{vs}/C_{vp}	Systemic/pulmonary venous compliance
E_{maxl}	Left maximum time-varying elastance
i, u	Current, voltage
k	Coupling factor for coronary model
L	Arterial inertance
L_s/L_p	Systemic/pulmonary inertance
LV/RV	Left/right ventricle
LVAD	Left Ventricular Assist Device
LVEF	Left ventricular variable-elastance function block
P_{as}/p_{ap}	Arterial systemic/pulmonary pressure
P_h, P_b	Polarization
p_{la}/p_{ra}	Left/right atrial pressure
p_{lv}/p_{rv}	Left ventricular pressure
p_t	Intra-thoracic pressure
p_{vs}/p_{vp}	Systemic/pulmonary venous pressure
Q_{cor}	Coronary flow
R_{as}/R_{ap}	Systemic/pulmonary arterial resistance
R_c	Characteristic resistance
R_{cor}	Total coronary resistance
R_{cs}/R_{cp}	Systemic/pulmonary characteristic resistance
$R_{it}(R_{in})$	Mitral (input) valve resistance
$R_{io}(R_{out})$	Aortic (output) valve resistance
R_p	Peripheral resistance
R_{ri}/R_{ro}	Right ventricular input/output valve
R_{vs}/R_{vp}	Systemic/pulmonary venous resistance
VCCS	Voltage-controlled current source
VCFS	Voltage-controlled flow source
VCVS	Voltage-controlled voltage source
Z_{in}	Input impedance

The investigations were done in the following conditions:

- Circulatory system in open loop configuration represented by left atrial pressure $p_{la} = 10$ mmHg and left ventricular end-systolic elastance $E_{\max} = 4$ mmHg/cm³.
- Resistance of a mitral input (R_{IN}) and aortic output (R_{OUT}) valve: $R_{IN} = R_{OUT} = 50 \text{ g} \cdot \text{s}^{-1} \cdot \text{cm}^{-3} \equiv 300 \Omega$.
- Left ventricular elastance function of the same shape [19] for each combination of the above mentioned artero-ventricular coupling (Fig. 3).
- Four-element standard Windkessel model of the systemic arterial circulation (R_cLR_pC) playing a role of ventricular load — Fig. 4: $R_c = 0.04$ mmHgscm⁻³ — characteristic resistance of the aorta, $L = 2 \cdot 10^{-3}$ mmHg · s² · cm⁻³ — arterial inertance, $R_p = 0.93$ mmHg · s · cm⁻³ — peripheral resistance, $C = 2.4$ cm³/mmHg — arterial compliance.

The tests performed have evidenced that the time courses of the left ventricular and aortic pressures (Fig. 5) are independent of the design (numerical, physical or hybrid) of particular segments of the circulatory model; in spite of the model design, the corresponding time courses almost overlap. Big differences of maximum values of the ventricular and aortic pressures in systole are a result of rather large value of resistance representing the aortic valve.

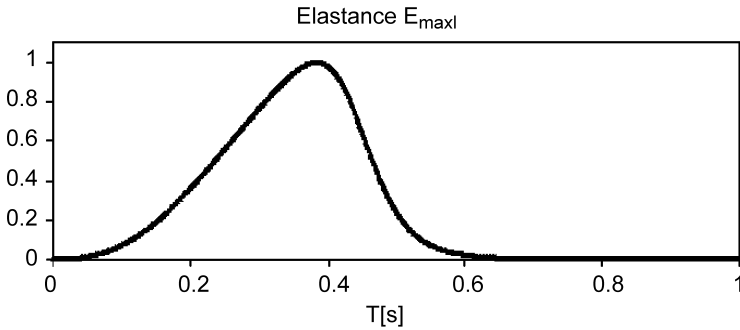


Fig. 3. Left ventricular systolic elastance function $E_{\max l}$ normalized to its maximum value [19]

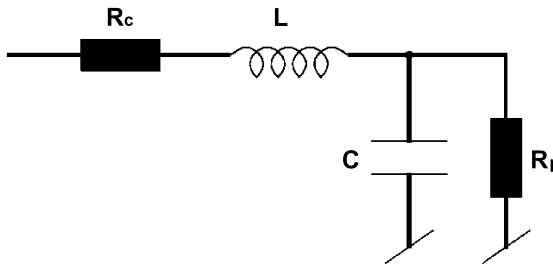


Fig. 4. Ventricular loading: standard four-element Windkessel (R_cLR_pC)

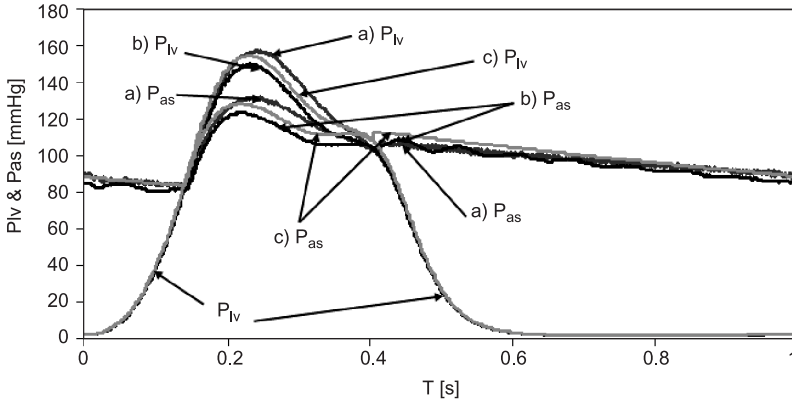


Fig. 5. Time courses of the left ventricular (p_{lv}) and aortic (p_{as}) pressures at different design (numerical, physical and hybrid) open-loop circulatory system: a) hybrid ventricle-physical (electrical) arterial model, b) hybrid ventricle-numerical arterial model, c) numerical ventricle-numerical arterial model

3.2. Effect of the Arterial System Models Structure on Selected Left Ventricular and Systemic Variables

Physical (electrical) lumped parameter models of the arterial system built up to play a role of the systemic arterial load of the left ventricle were designed in different structural combinations, using linear elements: resistors R , compliances C and inertances L (Table 2).

Table 2. Tests of structures of the arterial systemic model

Windkessel model	R_c [mmHg · scm ⁻³]	L [mmHg · s ² · cm ⁻³]	C [cm ³ /mmHg]	R_p [mmHg · s · cm ⁻³]
(a) Simplified three-element R_cCR_p	0.04	—	2.4	0.93
(b) Standard four-element R_cCLR_p	0.04	$2 \cdot 10^{-3}$	2.4	0.93
(c) Modified four-element R_cCLR_p	0.04	$5 \cdot 10^{-3}$	2.4	0.93
(d) Three-lump “ladder” Windkessel	$R_1 = 4.3 \cdot 10^{-3}$ $R_2 = 12.8 \cdot 10^{-3}$ $R_3 = 17.1 \cdot 10^{-3}$	$L_1 = 2.8 \cdot 10^{-4}$ $L_2 = 8.57 \cdot 10^{-4}$ $L_3 = 11 \cdot 10^{-4}$	$C_1 = 1.37$ $C_2 = 1.2$ $C_3 = 0.43$	0.93

Schematic representations of the arterial model structures are shown in Fig. 6. During the investigations, three of them: a, b and c (electrical) were connected to the hybrid (numerical-electrical) left ventricle and the last one, numerical (d) – to the numerical ventricle.

Laboratory tests were performed to verify how the structure of the systemic loading affects the left ventricular and aortic pressures in the open loop circulatory configuration. The obtained results, in the form of time courses, are reported in Fig. 7.

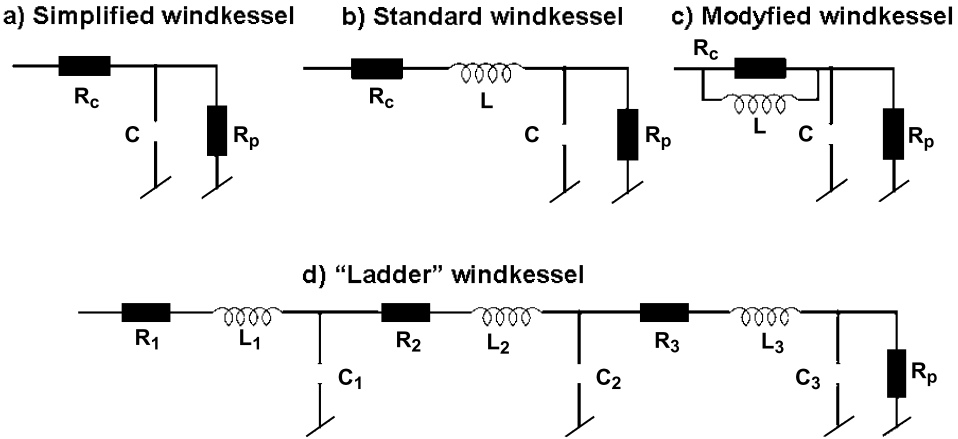


Fig. 6. Different structures of the systemic arterial tree model

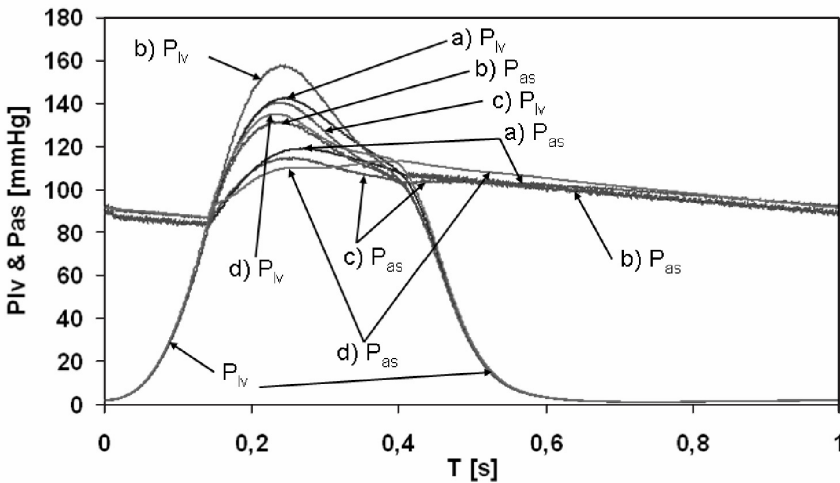


Fig. 7. Time courses of the left ventricular (p_{lv}) and aortic (p_{as}) pressures with the arterial load: a) simplified Windkessel, b) standard Windkessel, c) modified Windkessel, d) "ladder" Windkessel. Open loop configuration

4. Conclusions

1. The investigations performed evidenced that the left ventricular (p_{lv}) and aortic (p_{as}) pressures, in the open loop circulatory model, practically do not depend on the design (numerical, physical or hybrid) of the left ventricle and systemic arterial tree coupling (see paragraph 3.1) if its structure remains unchanged. The corresponding transient characteristics almost overlap.

2. When the systemic arterial model structures are different (see paragraph 3.2), the result achieved with the standard Windkessel ventricular load is significantly different from those obtained with other arterial model structures; in this case maximum systolic ventricular and aortic pressures are much bigger.

3. In the case of an “oscillatory” configuration, built up of the hybrid (numerical-electrical) ventricle and the aorta with the standard arterial Windkessel model (both numerical), it is enough to connect a relatively small capacitor to the left ventricular output to get rid of the oscillations.

4. The configuration based on the hybrid (numerical-electrical) left ventricle and the aorta with the arterial modified Windkessel model (both electrical) is an interesting solution thanks to the parallel connection of the inertance and characteristic resistance of the aorta, limiting both a fold resembling a “dicrotic notch” and the kinetic energy dissipation accumulated in the inertance; in this case, the model “aortic” pressure time course is very similar to the natural one.

5. The “ladder” type peripheral load can be successfully applied thanks to a big value of compliance C_1 that “separates” in some way relatively small inertance L_1 from larger inertances L_2 and L_3 .

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